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Derivation of and Discussions on the Forward-Looking Radar Imaging Point Spread Function

by DaHan Liao

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Army Research Laboratory

Adelphi, MD 20783-1138

ARL-TN-0624   **August 2014**

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.
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1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
August 2014	Final	2013–2014

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Derivation of and Discussions on the Forward-Looking Radar Imaging Point Spread Function	
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)	5d. PROJECT NUMBER
DaHan Liao	
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
U.S. Army Research Laboratory ATTN: RDRL-SER-U 2800 Powder Mill Road Adelphi, MD 20783-1138	ARL-TN-0624

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Efficient closed-form expressions for the point spread function are established for a forward-looking, ultra-wideband, near-field imaging algorithm that exploits a coherent frequency domain technique for focusing. The imaging performance (resolution, sidelobe level, ambiguous artifact level, etc.) is analyzed for on-surface and subsurface targets and compared for both single- and multi-aperture sensing geometries. The closed-form formulations and observations put forth herein are applicable for a standoff sensing system employing a two-end-transmitter bistatic configuration; the theoretical results overall, however, can be readily extended for other standard sensing scenarios, including monostatic and fully multistatic ones.

15. SUBJECT TERMS
Computational electromagnetics, electromagnetic scattering, radar Imaging

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE	UU	16	DaHan Liao
Unclassified	Unclassified	Unclassified			19b. TELEPHONE NUMBER *(Include area code)* 301-394-1741

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18

Contents

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1. Introduction

The point spread function (PSF) is the image response of an imaging system in sensing a scene containing a single point scatterer. As an extended scatterer can be approximated as a collection of point scatterers in many cases, the PSF is useful for characterizing the image response in generalized sensing scenarios. The PSF then provides a mathematical framework for quantifying the effects of the sensing geometry and associated parameters—such as aperture configuration (single- vs. multi-aperture), array aperture size, spectral bandwidth, spatial and spectral sampling interval, etc.—on the imaging and detection performance. Specifically, the PSF offers guidance on 1) the design and optimization the imaging system, 2) the prediction of the detectability limit of the system, and 3) the interpretation of the scene image response.

In this work, the PSF is studied within the context of ultra-wideband (UWB), electromagnetic-oriented standoff target localization. The twofold objective here is to first derive efficient, simplified closed-form approximations for the single- and multi-aperture forward-looking PSF for on-surface and subsurface scatterers and then, using those approximations, put forth observations useful for understanding the imaging results. Although the focus of the study is on a sensing setup that emulates the operation of an existing forward-looking radar system, it is expected the derivations featured can be generalized for the treatment of a variety of standoff sensing configurations.

2. Analytical Formulations

From the time-reversal-based imaging functional, the PSF is first formulated for an on-surface scatterer and then the derivation is extended for a subsurface scatterer.

2.1 On-Surface Scatterer

The sensing geometry treated here is akin to the one studied in Reference 1. Consider an array of vertically polarized point radiators and a near-ground or on-surface point scatterer at $\vec{r}_s = (r_s, \theta_s, \phi_s)$, approximating the Green's function for the interactions of the co-polarized field component as[2]

$$G(\vec{r}, \vec{r}', k) \propto e^{-ik|\vec{r}-\vec{r}'|} \left\{ \frac{1}{|\vec{\rho}|} + \frac{R(\theta_{illu})}{|\vec{\rho}|} + \frac{C_1}{|\vec{\rho}|^2} + \frac{C_2}{|\vec{\rho}|^3} + ... \right\} = e^{-ik|\vec{r}-\vec{r}'|} F(\vec{r}, \vec{r}'), \qquad (1)$$

and assuming observation with an array of isotropic point receivers, the PSF for a single-aperture configuration (positioned above the ground at height $z_T = z_R$) can be constructed from the time-reversal-based imaging functional as[3]

$$PSF_{SA}(\vec{r},\vec{r}_s) \approx \sum_k \sum_{L_R} \sum_{L_T} e^{ik|\vec{r}_s-\vec{r}_T|} F(\vec{r}_s,\vec{r}_T) e^{ik|\vec{r}_R-\vec{r}_s|} F(\vec{r}_R,\vec{r}_s) e^{-ik|\vec{r}-\vec{r}_T|} F(\vec{r},\vec{r}_T) e^{-ik|\vec{r}_R-\vec{r}|} F(\vec{r}_R,\vec{r}). \quad (2)$$

Using far-field expressions for the distance vectors, the $F(\cdot)$ terms can be moved outside the summation operation, and orienting the transmitting and receiving arrays parallel to the y-axis ($x_T = x_R = 0$), the function in Eq. 2 becomes

$$PSF_{SA}(\vec{r},\vec{r}_s) \propto F(\vec{r}_s,\vec{r}_T) F(\vec{r}_R,\vec{r}_s) F(\vec{r},\vec{r}_T) F(\vec{r}_R,\vec{r}) \cdot$$
$$\sum_{n_k} \sum_{n_R} \sum_{n_T} e^{2in_k\delta k(r_s-r+z_T(\cos\theta-\cos\theta_s))} e^{i(n_k\delta k+k_c)(n_R\delta y_R+n_T\delta y_T)(\sin\theta\sin\phi-\sin\theta_s\sin\phi_s)}, \quad (3)$$

where δk, δy_R, and δy_T are the k-space sampling interval, the receiver spacing, and the transmitter spacing, respectively; k_c is the center wavenumber; the wavenumber bandwidth is then $\Delta k = N_k \delta k$, and the aperture sizes are $L_R = N_R \delta y_R$ and $L_T = N_T \delta y_T$. Applying a stationary-phase-like approach to evaluate the summation in k-space and noting the relation

$$\left| \sum_{n=0}^{N-1} e^{-inu} \right| = \left| \frac{\sin\left(\frac{Nu}{2}\right)}{\sin\left(\frac{u}{2}\right)} \right|, \quad (4)$$

the amplitude of Eq. 3 approximately simplifies to

$$\left| PSF_{SA}(\vec{r},\vec{r}_s) \right| \propto \left| F(\vec{r}_s,\vec{r}_T) F(\vec{r}_R,\vec{r}_s) F(\vec{r},\vec{r}_T) F(\vec{r}_R,\vec{r}) \right| \cdot$$
$$\left| \frac{\sin\left(\Delta k\left(r_s-r+z_T(\cos\theta-\cos\theta_s)\right)\right)}{\sin\left(\delta k\left(r_s-r+z_T(\cos\theta-\cos\theta_s)\right)\right)} \cdot \frac{\sin\left(\frac{L_R k_c(\sin\theta\sin\phi-\sin\theta_s\sin\phi_s)}{2}\right)}{\sin\left(\frac{\delta y_R k_c(\sin\theta\sin\phi-\sin\theta_s\sin\phi_s)}{2}\right)} \cdot \cos\left(\frac{L_T k_c(\sin\theta\sin\phi-\sin\theta_s\sin\phi_s)}{2}\right) \right|, \quad (5)$$

where it is assumed that there is a transmitter at each end of the aperture L_T. The first $\sin(\cdot)/\sin(\cdot)$ term in Eq. 5 dictates the behavior of the PSF in range: on the xy-plane ($\theta = \theta_s = \pi/2$), it can be shown that the first null occurs at $c/2\Delta f$—which is the resolution in range, δr, independent of r_s and ϕ_s. The periodicity of this term determines the frequency sampling interval needed: for maximum range $r_{s,max}$, the condition $\delta f < c/2r_{s,max}$ must be met. Similarly, the cross-range properties of the PSF are controlled by the last two terms in Eq. 5. For the current two-transmitter system (and $L_T = L_R$), based on the location of the first null of these two terms, it can be readily seen that the angular resolution depends on the angular position of the scatterer as

$$\delta\phi_{SA} = \phi_s - \sin^{-1}\left(\sin\phi_s - \frac{\lambda_c}{2L_T}\right), \quad 0 \le \phi_s \le \frac{\pi}{2}. \tag{6}$$

At $\phi_s = 0$, $\delta\phi_{SA} = \lambda_c / 2L_T$; the angular resolution degrades as the scatterer moves away from the broadside direction ($\phi_s = 0$) of the aperture. In general, the spatial sampling should be less than $\lambda_{min}/2$; however, for an UWB system, it has been shown that high cross-range resolution can be achieved with coarser sampling than the stated criterion.[4]

For transmitters and receivers with symmetric patterns about the array axis (or, in this case, the *y*-axis), each scatterer would give rise to two identical image responses—one at the true location of the scatterer (at ϕ_s) and an ambiguous component at the location that is mirrored about the array axis (at $\pi - \phi_s$). The ambiguous component can be suppressed through the use of the multi-aperture sensing geometry—described below—or by first employing transmitters and receivers with forward directivity and then explicitly incorporating the antenna pattern functions into the imaging reconstruction. Both of these methods would reduce the mirrored image components originated from targets positioned in the backside of the array, which can mask target responses in the forward-looking image.

Following a line of derivations similar to that summarized above, the approximate PSF for the multi-aperture configuration with interweaved left and right excitation scheme can be found as

$$|PSF_{MA}(\vec{r},\vec{r}_s)| \propto |F(\vec{r}_s,\vec{r}_T)F(\vec{r}_R,\vec{r}_s)F(\vec{r},\vec{r}_T)F(\vec{r}_R,\vec{r})| \cdot$$

$$\left| \frac{\sin\left(\Delta k\left(r_s - r + z_T(\cos\theta - \cos\theta_s) + \frac{\delta x}{4}(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s)\right)\right)}{\sin\left(\delta k\left(r_s - r + z_T(\cos\theta - \cos\theta_s) + \frac{\delta x}{4}(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s)\right)\right)} \right| \cdot$$

$$\frac{\sin\left(\frac{L_R k_c(\sin\theta\sin\phi - \sin\theta_s\sin\phi_s)}{2}\right)}{\sin\left(\frac{\delta y_R k_c(\sin\theta\sin\phi - \sin\theta_s\sin\phi_s)}{2}\right)} \cdot \cos\left(\frac{L_T k_c(\sin\theta\sin\phi - \sin\theta_s\sin\phi_s)}{2} - \frac{\delta x k_c(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s)}{2}\right) \cdot$$

$$\left. \frac{\sin\left(L_x k_c(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s)\right)}{\sin\left(\delta x k_c(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s)\right)} \right|, \tag{7}$$

in which L_x is the forward-traveling distance and δx the separation between successive left or right apertures ($L_x \approx N_x \delta x$). For the set of parameters commonly employed in forward-looking studies, in determining the range and cross-range imaging behaviors, the effect of the terms $\delta x(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s)/4$ and $\delta x k_c(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s)/2$ can be considered to be

negligible. As such, the frequency sampling criterion and range resolution are the same as those for the single-aperture case. The cross-range resolution is dependent on the angular position of the scatterer as before; specifically, again on the *xy*-plane, $\delta\phi_{MA} = \min\left(\delta\phi_{SA}, \delta\phi_{L_x}\right)$, with

$$\delta\phi_{L_x} = \cos^{-1}\left(\cos\phi_s - \frac{\lambda_c}{2L_x}\right) - \phi_s, \quad 0 \le \phi_s \le \frac{\pi}{2}. \tag{8}$$

For $L_x > L_T$ (e.g., $L_x = 10$ m and $L_T = 2$ m), higher angular resolution occurs over the region *away* from the broadside direction of the aperture: at $\phi_s = 0$, $\delta\phi_{MA} = \lambda_c / 2L_T$, and the highest resolution is reached at $\phi_s = \pi/2$, with $\delta\phi_{MA} = \lambda_c / 2L_x$.

The resolution in θ of both the single- and multi-aperture cases can be deduced based on the first null of the first $\sin(\cdot)/\sin(\cdot)$ term in Eqs. 5 and 7. Upon setting $r = r_s$, it is found that

$$\delta\theta_{SA} = \delta\theta_{MA} = \Delta\lambda / 2z_T, \tag{9}$$

where $\Delta\lambda = c / \Delta f$. The above result stipulates z_T is relatively large (e.g., $z_T = 2$ m); otherwise, the other terms within the PSF would also need to be taken into account. Eq. 9 gives the resolution in θ with respect to the coordinate system origin; a more appropriate definition for the resolution in elevation can be derived with respect to the aperture center. Designating θ' as the elevation angle with respect to the aperture center, for a multi-aperture configuration, with the condition that $\theta'_s \ge \pi/2$ but located in the vicinity of the *xy*-plane, it can be shown that $\delta\theta'_{MA} = \min(\delta\theta'_{L_T}, \delta\theta'_{L_x})$, in which

$$\delta\theta'_{L_T} = \pi - \theta'_s - \sin^{-1}\left(\sin\theta'_s - \frac{\lambda_c}{2L_T \sin\phi_s}\right), \quad 0 < \phi_s \le \frac{\pi}{2}; \tag{10}$$

$$\delta\theta'_{L_x} = \pi - \theta'_s - \sin^{-1}\left(\sin\theta'_s - \frac{\lambda_c}{2L_x \cos\phi_s}\right), \quad 0 \le \phi_s < \frac{\pi}{2}. \tag{11}$$

The above supposes the terms of the PSF in θ' corresponding to the L_T and L_x apertures have definable nulls; if only one of these nulls is definable, then that null is used to determine the resolution. Thus, for instance, at $\phi_s = 0$, $\delta\theta'_{MA} = \delta\theta'_{L_x}$, and at $\phi_s = \pi/2$, $\delta\theta'_{MA} = \delta\theta'_{L_T}$. These results can be reinterpreted for the single-aperture case. Specifically, it is seen that at $\phi_s = 0$, the single-aperture case does not have resolution in θ', and at $0 < \phi_s \le \pi/2$, $\delta\theta'_{SA} = \delta\theta'_{L_T}$ —if a null is definable.

A note of caution needs to be added with regard to the suppression of the imaging sidelobes in range. In the setup of the current problem, surface clutter close to the transmitter generates sidelobes in range that are especially detrimental to imaging performance. This is due to the fact that scatterers local to the transmitter not only produce the strongest responses but also create

sidelobes that spread out in all azimuth directions. Because of the fast-decaying nature of the backscattering response from a down-range scatterer (that is, asymptotically, its normalized backscattering amplitude is of the form $\propto 1/r^2$), judicious selection of both δf and a frequency domain window is necessary to ensure the sidelobes of the near-range surface clutter are weaker than the response from the down-range scatterer. It is seen that this is particularly critical for sensing with horizontal polarization since—as evident in the imaging results[1]—scatterers have a weaker response at horizontal polarization than at vertical polarization. According to Eqs. 5 and 7, the sidelobes of the near-range surface clutter should fall off as $\propto 1/r$; however, their dependence on distance in the imaging algorithm as used in this study is actually $\propto r$—as a result of the application of additional amplitude normalization terms needed to translate the imaging intensities into radar cross section (RCS) values. In view of the above observations, a Blackman window with $\alpha = 0.08$[5] is chosen and imposed on the frequency domain data. A desirable compromise among main lobe width, sidelobe amplitude, and sidelobe decay rate is achieved with this window. Consistent and acceptable results are obtained even though the sidelobes of the near-range clutter—after windowing—effectively fall off only as $\propto 1/r$—a decay not as fast as the aforementioned $\propto 1/r^2$ behavior of the response from a down-range scatterer. Theoretically, the range sidelobes from a surface scatterer (and its first periodic image component) can also be reduced by decreasing δf beyond the required value of $\delta f < c / 2r_{s,max}$; however, it is seen that this approach only produces results with limited quality. Better performance and higher algorithm efficiency are realized with the windowing method.

Also in order are a few notes to summarize the important differences between the single- and multi-aperture cases—in terms of imaging performance as conveyed by the PSF formulated. As observed with the set of parameters applied herein, multi-aperture sensing leads to lower sidelobe levels in the cross-range and elevation directions; however, the range sidelobe levels and range resolution are comparable to those of the single-aperture case. In the cross-range direction, the multi-aperture configuration gives better resolution for scatterers situated away from the broadside direction and enables attenuation of the ambiguous image components. (For the multi-aperture case, the ambiguous component for each target now manifests as a series of peaks—with each peak corresponding to one aperture. Hence, the main-lobe-to-ambiguous-response ratio is $2N_x$.) In the θ direction, the resolution is the same for the two configurations. In the θ' direction, better resolution is noted with multi-aperture imaging for scatterers positioned at azimuth angles away from $\phi = \pm \pi / 2$, toward broadside.

2.2 Subsurface Scatterer

Equations 5 and 7 are applicable for on-surface targets. Employing the appropriate Green's function from Reference 2 for subsurface observation points, the PSF for a shallow-buried point target—for the multi-aperture case—can be shown to be of the same form as Eq. 7, except the first $\sin(\cdot)/\sin(\cdot)$ term must be modified as

$$\left| \frac{\sin\left(\Delta k \left(r_s - r + k'_{1z,s} \mid z_s \mid - k'_{1z} \mid z \mid + z_T \left(\cos\theta - \cos\theta_s \right) + \frac{\delta x}{4} \left(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s \right) \right) \right)}{\sin\left(\delta k \left(r_s - r + k'_{1z,s} \mid z_s \mid - k'_{1z} \mid z \mid + z_T \left(\cos\theta - \cos\theta_s \right) + \frac{\delta x}{4} \left(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s \right) \right) \right)} \right|, \quad (12)$$

where

$$k'_{1z,s} = \left(\varepsilon^c_{r,g}(f_c) - 1 + \left(\frac{z_T}{r_s} \right)^2 \right)^{\frac{1}{2}}; \quad (13)$$

$$k'_{1z} = \left(\varepsilon^c_{r,g}(f_c) - 1 + \left(\frac{z_T}{r} \right)^2 \right)^{\frac{1}{2}}; \quad (14)$$

and $\varepsilon^c_{r,g}(f_c)$ is the complex relative dielectric constant of the ground (at the center frequency). Similarly, the first $\sin(\cdot)/\sin(\cdot)$ term of Eq. 5 can be replaced with Eq. 12 sans the $\delta x \left(\sin\theta\cos\phi - \sin\theta_s\cos\phi_s \right)/4$ term to obtain the subsurface, single-aperture PSF. Using these subsurface functionals, for sensing and imaging at the grazing-angle regime, it can be shown that the frequency sampling criterion and the range and cross-range resolutions follow the same standard forms as previously established for the on-surface case, while the resolution in θ approximately takes the form

$$\delta\theta_{SA} = \delta\theta_{MA} = \frac{\Delta\lambda}{2\left(\left(\varepsilon^c_{r,g}(f_c) - 1 \right)^{\frac{1}{2}} r_s + z_T \right)}, \quad (15)$$

where, for convenience, $\varepsilon^c_{r,g}(f_c)$ should be estimated with only its real part—assuming the imaginary part is relatively small. A closed-form expression for the resolution in θ' for the subsurface case, however, cannot be so readily obtained. A more meaningful numerical analysis of the resolution in elevation (or in depth) can also be reached by inspecting the behaviors of the exact PSF along constant phase lines below the ground.

3. Conclusions

Analytical closed-form formulations for the single- and multi-aperture PSF of a forward-looking radar imaging system are derived in this work for both on-surface and subsurface point scatterers. It is shown that the multi-aperture configuration has reduced cross-range sidelobe levels and better cross-range resolution (away from the broadside, or forward, direction) as compared to the single-aperture configuration, and the ambiguous image component in the

azimuth direction can be suppressed by an integration of the multi-perspective scattering data. However, the sidelobe levels and resolution in the range direction are the same for both the single- and multi-aperture cases. The approximate expressions presented herein are appropriate for a bistatic sensing configuration in which the excitation is supplied by the two transmitters located at the ends of a linear array—a sensing geometry that is consistent with the one employed by an existing forward-looking radar setup; nevertheless, the derivations follow a framework that can be applied to other sensing configurations not explicitly treated in this work.

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